# Package: brglm2 (via r-universe)

September 12, 2024

Title Bias Reduction in Generalized Linear Models

Version 0.9.2

**Description** Estimation and inference from generalized linear models based on various methods for bias reduction and maximum penalized likelihood with powers of the Jeffreys prior as penalty. The 'brglmFit' fitting method can achieve reduction of estimation bias by solving either the mean bias-reducing adjusted score equations in Firth (1993) <doi:10.1093/biomet/80.1.27> and Kosmidis and Firth (2009) <doi:10.1093/biomet/asp055>, or the median bias-reduction adjusted score equations in Kenne et al. (2017) <doi:10.1093/biomet/asx046>, or through the direct subtraction of an estimate of the bias of the maximum likelihood estimator from the maximum likelihood estimates as in Cordeiro and McCullagh (1991) <https://www.jstor.org/stable/2345592>. See Kosmidis et al (2020) <doi:10.1007/s11222-019-09860-6> for more details. Estimation in all cases takes place via a quasi Fisher scoring algorithm, and S3 methods for the construction of of confidence intervals for the reduced-bias estimates are provided. In the special case of generalized linear models for binomial and multinomial responses (both ordinal and nominal), the adjusted score approaches to mean and media bias reduction have been found to return estimates with improved frequentist properties, that are also always finite, even in cases where the maximum likelihood estimates are infinite (e.g. complete and quasi-complete separation; see Kosmidis and Firth, 2020 <doi:10.1093/biomet/asaa052>, for a proof for mean bias reduction in logistic regression).

## URL https://github.com/ikosmidis/brglm2

BugReports https://github.com/ikosmidis/brglm2/issues

**Depends** R (>= 3.3.0)

**Imports** MASS, stats, Matrix, graphics, nnet, enrichwith, numDeriv **License** GPL-3

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Encoding UTF-8 LazyData true RoxygenNote 7.2.3 Roxygen list(markdown = TRUE)

Suggests detectseparation, knitr, rmarkdown, covr, tinytest, VGAM, brglm, mbrglm

VignetteBuilder knitr

Repository https://ikosmidis.r-universe.dev

RemoteUrl https://github.com/ikosmidis/brglm2

RemoteRef HEAD

RemoteSha 60a8c7bb46f468f36bbfc7fe37e6cdb4805e5d85

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aids

The effects of AZT in slowing the development of AIDS symptoms

# Description

The data is from a 3-year study on the effects of AZT in slowing the development of AIDS symptoms. 338 veterans whose immune systems were beginning to falter after infection with the AIDS virus were randomly assigned either to receive AZT immediately or to wait until their T cells showed severe immune weakness.

# Usage

aids

#### Format

A data frame with 4 rows and 4 variables:

- symptomatic: counts of veterans showing AIDS symptoms during the 3-year study
- asymptomatic: counts of veterans not showing AIDS symptoms during the 3-year study
- race: the race of the veterans with levels "White" and "Black"
- AZT\_use: whether the veterans received AZT immediately ("Yes") or waited until their T cells showed severe immune weakness ("No")

#### Source

The data set is analyzed in Agresti (2002, Subsection 5.4.2). Its original source is New York Times, Feb. 15, 1991.

## References

Agresti A (2002). Categorical Data Analysis. Wiley Series in Probability and Statistics. Wiley.

#### See Also

brmultinom()

alligators

## Description

Alligator food choice data

#### Usage

alligators

#### Format

A data frame with 80 rows and 5 variables:

- foodchoice: primary food type, in volume, found in an alligator's stomach, with levels fish, invertebrate, reptile, bird, other
- lake: lake of capture with levels Hancock, Oklawaha, Trafford, George.
- gender: gender of the alligator with levels Male and Female
- size: size of the alligator with levels <=2.3 meters long and >2.3 meters long
- freq: number of alligators for each foodchoice, lake, gender and size combination

#### Source

The alligators data set is analyzed in Agresti (2002, Subsection 7.1.2).

## References

Agresti A (2002). Categorical Data Analysis. Wiley Series in Probability and Statistics. Wiley.

## See Also

brmultinom()

bracl

Bias reduction for adjacent category logit models for ordinal responses using the Poisson trick.

## Description

bracl() is a wrapper of brglmFit() that fits adjacent category logit models with or without proportional odds using implicit and explicit bias reduction methods. See Kosmidis & Firth (2011) for details. bracl

# Usage

```
bracl(
  formula,
  data,
  weights,
  subset,
  na.action,
  parallel = FALSE,
  contrasts = NULL,
  model = TRUE,
   x = TRUE,
   control = list(...),
  ...
)
```

# Arguments

formula	a formula expression as for regression models, of the form response ~ predictors. The response should be a factor (preferably an ordered factor), which will be interpreted as an ordinal response, with levels ordered as in the factor. The model must have an intercept: attempts to remove one will lead to a warning and be ignored. An offset may be used. See the documentation of formula for other details.
data	an optional data frame, list or environment in which to interpret the variables occurring in formula.
weights	optional case weights in fitting. Default to 1.
subset	expression saying which subset of the rows of the data should be used in the fit. All observations are included by default.
na.action	a function to filter missing data.
parallel	if FALSE (default), then a non-proportional odds adjacent category model is fit, assuming different effects per category; if TRUE then a proportional odds adjacent category model is fit. See Details.
contrasts	a list of contrasts to be used for some or all of the factors appearing as variables in the model formula.
model	logical for whether the model matrix should be returned.
x	should the model matrix be included with in the result (default is TRUE).
control	a list of parameters for controlling the fitting process. See brglmControl() for details.
	arguments to be used to form the default control argument if it is not supplied directly.

# Details

The bracl() function fits adjacent category models, which assume multinomial observations with probabilities with proportional odds of the form

bracl

$$\log \frac{\pi_{ij}}{\pi_{ij+1}} = \alpha_j + \beta^T x_i$$

or with non-proportional odds of the form

$$\log \frac{\pi_{ij}}{\pi_{ij+1}} = \alpha_j + \beta_j^T x_i$$

where  $x_i$  is a vector of covariates and  $\pi_{ij}$  is the probability that category j is observed at the covariate setting i.

## Author(s)

Ioannis Kosmidis [aut, cre] <ioannis.kosmidis@warwick.ac.uk>

#### References

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Agresti, A (2010). *Analysis of Ordinal Categorical Data* (2nd edition). Wiley Series in Probability and Statistics. Wiley.

Albert A, Anderson J A (1984). On the Existence of Maximum Likelihood Estimates in Logistic Regression Models. *Biometrika*, **71**, 1-10. doi:10.2307/2336390.

Kosmidis I, Firth D (2011). Multinomial logit bias reduction via the Poisson log-linear model. *Biometrika*, **98**, 755-759. doi:10.1093/biomet/asr026.

Palmgren J (1981). The Fisher Information Matrix for Log Linear Models Arguing Conditionally on Observed Explanatory Variables. *Biometrika*, **68**, 563-566. doi:10.1093/biomet/68.2.563.

#### See Also

nnet::multinom(), brmultinom()

## Examples

```
data("stemcell", package = "brglm2")
```

brglm2

## Description

Estimation and inference from generalized linear models using implicit and explicit bias reduction methods (Kosmidis, 2014), and other penalized maximum likelihood methods. Currently supported methods include the mean bias-reducing adjusted scores approach in Firth (1993) and Kosmidis & Firth (2009), the median bias-reduction adjusted scores approach in Kenne Pagui et al. (2017), the correction of the asymptotic bias in Cordeiro & McCullagh (1991), the mixed bias-reduction adjusted scores approach likelihood with powers of the Jeffreys prior as penalty, and maximum likelihood.

## Details

In the special case of generalized linear models for binomial, Poisson and multinomial responses (both nominal and ordinal), mean and median bias reduction and maximum penalized likelihood return estimates with improved frequentist properties, that are also always finite, even in cases where the maximum likelihood estimates are infinite (e.g. complete and quasi-complete separation in multinomial regression). Estimation in all cases takes place via a modified Fisher scoring algorithm, and S3 methods for the construction of confidence intervals for the reduced-bias estimates are provided.

The core model fitters are implemented by the functions brglm\_fit() (univariate generalized linear models), brmultinom() (baseline category logit models for nominal multinomial responses), bracl() (adjacent category logit models for ordinal multinomial responses), and brnb() for negative binomial regression.

The similarly named **brglm** R package can only handle generalized linear models with binomial responses. Special care has been taken when developing **brglm2** in order not to have conflicts when the user loads **brglm2** and **brglm** simultaneously. The development and maintenance of the two packages will continue in parallel, until **brglm2** incorporates all **brglm** functionality and provides an appropriate wrapper to the **brglm::brglm()** function.

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#### References

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, **108**, 71-82. doi:10.1093/biomet/asaa052.

Cordeiro G M, McCullagh P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B (Methodological)*, **53**, 629-643. doi:10.1111/j.25176161.1991.tb01852.x.

Firth D (1993). Bias reduction of maximum likelihood estimates, Biometrika, **80**, 27-38. doi:10.2307/2336755.

Kenne Pagui E C, Salvan A, Sartori N (2017). Median bias reduction of maximum likelihood estimates. *Biometrika*, **104**, 923–938. doi:10.1093/biomet/asx046.

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Kosmidis I, Firth D (2009). Bias reduction in exponential family nonlinear models. *Biometrika*, **96**, 793-804. doi:10.1093/biomet/asp055.

Kosmidis I, Firth D (2010). A generic algorithm for reducing bias in parametric estimation. *Electronic Journal of Statistics*, **4**, 1097-1112. doi:10.1214/10EJS579.

Kosmidis I (2014). Bias in parametric estimation: reduction and useful side-effects. *WIRE Computational Statistics*, **6**, 185-196. doi:10.1002/wics.1296.

## See Also

brglm\_fit(), brmultinom(), bracl()

brglm2-defunct Defunct Functions in package brglm2

#### Description

The functions or variables listed here are no longer part of brglm2.

#### Usage

```
check_infinite_estimates(...)
```

detect\_separation(...)

#### Arguments

... arguments to be passed to functions and methods.

#### Details

- detect\_separation(): This function is defunct from **brglm2** since version 0.8.0. A new version of detect\_separation() is now maintained in the **detectseparation** R package.
- check\_infinite\_estimates() is defunct from **brglm2** since version 0.8.0. An new version of check\_infinite\_estimates() is now maintained in the **detectseparation** R package.

brglmControl

## Description

Typically only used internally by brglmFit(), but may be used to construct a control argument.

# Usage

```
brglmControl(
  epsilon = 1e-06,
 maxit = 100,
  check_aliasing = TRUE,
  trace = FALSE,
 type = c("AS_mixed", "AS_mean", "AS_median", "correction", "MPL_Jeffreys", "ML"),
  transformation = "identity",
  slowit = 1,
  response_adjustment = NULL,
 max_step_factor = 12,
 a = 1/2,
  . . .
)
brglm_control(
  epsilon = 1e-06,
 maxit = 100,
 check_aliasing = TRUE,
  trace = FALSE,
 type = c("AS_mixed", "AS_mean", "AS_median", "correction", "MPL_Jeffreys", "ML"),
  transformation = "identity",
  slowit = 1,
  response_adjustment = NULL,
 max_step_factor = 12,
 a = 1/2,
  . . .
)
```

## Arguments

epsilon	positive convergence tolerance epsilon. Default is 1e-06.
maxit	integer giving the maximal number of iterations allowed. Default is 100.
check_aliasing	logical indicating where a QR decomposition of the model matrix should be used to check for aliasing. Default is TRUE. See Details.
trace	logical indicating if output should be produced for each iteration. Default is FALSE.

type	the type of fitting method to be used. The options are "AS_mean" (mean-bias re-					
	ducing adjusted scores), "AS_median" (median-bias reducing adjusted scores),					
	"AS_mixed" (bias reduction using mixed score adjustments; default), "correction"					
	(asymptotic bias correction), "MPL_Jeffreys" (maximum penalized likelihood with powers of the Jeffreys prior as penalty) and "ML" (maximum likelihood).					
transformation	the transformation of the dispersion to be estimated. Default is "identity". See Details.					
slowit	a positive real used as a multiplier for the stepsize. The smaller it is the smaller					
	the steps are. Default is 1.					
response_adjustment						
	a (small) positive constant or a vector of such. Default is NULL. See Details.					
<pre>max_step_factor</pre>						
	the maximum number of step halving steps to consider. Default is 12.					
а	power of the Jeffreys prior penalty. See Details.					
	further arguments passed to brglmControl(). Currently ignored in the output.					

## Details

brglmControl() provides default values and sanity checking for the various constants that control the iteration and generally the behaviour of brglmFit().

When trace = TRUE, calls to cat() produce the output for each iteration. Hence, options(digits = \*) can be used to increase the precision.

When check\_aliasing = TRUE (default), a QR decomposition of the model matrix is computed to check for aliasing. If the model matrix is known to be of full rank, then check\_aliasing = FALSE avoids the extra computational overhead of an additional QR decomposition, which can be substantial for large model matrices. However, setting check\_aliasing = FALSE tells brglmFit() that the model matrix is full rank, and hard to trace back errors will result if it is rank deficient.

transformation sets the transformation of the dispersion parameter for which the bias reduced estimates are computed. Can be one of "identity", "sqrt", "inverse", "log" and "inverseSqrt". Custom transformations are accommodated by supplying a list of two expressions (transformation and inverse transformation). See the examples for more details.

The value of response\_adjustment is only relevant if brglmFit() is called with start = NULL, and family is binomial() or poisson(). For those models, an initial maximum likelihood fit is obtained on adjusted data to provide starting values for the iteration in brglmFit(). The value of response\_adjustment governs how the data is adjusted. Specifically, if family is binomial(), then the responses and totals are adjusted by response\_adjustment and 2 \* response\_adjustment, respectively; if family is poisson(), then the responses are adjusted by and response\_adjustment. response\_adjustment = NULL (default) is equivalent to setting it to "number of parameters" / "number of observations".

When type = "AS\_mixed" (default), mean bias reduction is used for the regression parameters, and median bias reduction for the dispersion parameter, if that is not fixed. This adjustment has been developed based on equivariance arguments (see, Kosmidis et al, 2020, Section 4) in order to produce regression parameter estimates that are invariant to arbitrary contrasts, and estimates for the dispersion parameter that are invariant to arbitrary non-linear transformations. type = "AS\_mixed" and type = "AS\_mean" return the same results if brglmFit() is called with family binomial() or poisson() (i.e. families with fixed dispersion).

#### brg1mContro1

When type = "MPL\_Jeffreys", brglmFit() will maximize the penalized log-likelihood

$$l(\beta,\phi) + a\log \det i(\beta,\phi)$$

where  $i(\beta, \phi)$  is the expected information matrix about the regression parameters  $\beta$  and the dispersion parameter  $\phi$ . See, vignette("iteration", "brglm2") for more information. The argument a controls the amount of penalization and its default value is a = 1/2, corresponding to maximum penalized likelihood using a Jeffreys-prior penalty. See, Kosmidis & Firth (2021) for proofs and discussion about the finiteness and shrinkage properties of the maximum penalized likelihood estimators for binomial-response generalized linear models.

The estimates from type = "AS\_mean" and type = "MPL\_Jeffreys" with a = 1/2 (default) are identical for Poisson log-linear models and logistic regression models, i.e. for binomial and Poisson regression models with canonical links. See, Firth (1993) for details.

brglm\_control() is an alias to brglmControl().

#### Value

a list with components named as the arguments, including symbolic expressions for the dispersion transformation (Trans) and its inverse (inverseTrans)

#### Author(s)

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#### References

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, **108**, 71-82. doi:10.1093/biomet/asaa052.

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Firth D (1993). Bias reduction of maximum likelihood estimates. Biometrika, **80**, 27-38. doi:10.2307/2336755.

#### See Also

brglm\_fit() and glm.fit()

# Examples

```
method = "brglmFit", transformation = "log")
coef(coalitionBR1, model = "dispersion")
## Just for illustration: Bias reduced estimation of dispersion^0.25
my_transformation <- list(expression(dispersion^0.25), expression(transformed_dispersion^4))
coalitionBRc <- update(coalitionBRi, transformation = my_transformation)
coef(coalitionBRc, model = "dispersion")</pre>
```

brglmFit

*Fitting function for* glm() *for reduced-bias estimation and inference* 

#### Description

brglmFit() is a fitting method for glm() that fits generalized linear models using implicit and explicit bias reduction methods (Kosmidis, 2014), and other penalized maximum likelihood methods. Currently supported methods include the mean bias-reducing adjusted scores approach in Firth (1993) and Kosmidis & Firth (2009), the median bias-reduction adjusted scores approach in Kenne Pagui et al. (2017), the correction of the asymptotic bias in Cordeiro & McCullagh (1991), the mixed bias-reduction adjusted scores approach in Kosmidis et al (2020), maximum penalized likelihood with powers of the Jeffreys prior as penalty, and maximum likelihood. Estimation is performed using a quasi Fisher scoring iteration (see vignette("iteration", "brglm2"), which, in the case of mean-bias reduction, resembles an iterative correction of the asymptotic bias of the Fisher scoring iterates.

## Usage

```
brglmFit(
  х,
 у,
 weights = rep(1, nobs),
  start = NULL,
 etastart = NULL,
 mustart = NULL,
 offset = rep(0, nobs),
  family = gaussian(),
  control = list(),
  intercept = TRUE,
  fixed_totals = NULL,
  singular.ok = TRUE
)
brglm_fit(
  х,
 у,
 weights = rep(1, nobs),
  start = NULL,
  etastart = NULL,
```

# brglmFit

```
mustart = NULL,
offset = rep(0, nobs),
family = gaussian(),
control = list(),
intercept = TRUE,
fixed_totals = NULL,
singular.ok = TRUE
)
```

#### Arguments

x	a design matrix of dimension n * p.
У	a vector of observations of length n.
weights	an optional vector of 'prior weights' to be used in the fitting process. Should be NULL or a numeric vector.
start	starting values for the parameters in the linear predictor. If NULL (default) then the maximum likelihood estimates are calculated and used as starting values.
etastart	applied only when start is not NULL. Starting values for the linear predictor to be passed to $glm.fit()$ when computing starting values using maximum likelihood.
mustart	applied only when start is not NULL. Starting values for the vector of means to be passed to glm.fit() when computing starting values using maximum likelihood.
offset	this can be used to specify an <i>a priori</i> known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases. One or more offset terms can be included in the formula instead or as well, and if more than one is specified their sum is used. See model.offset.
family	a description of the error distribution and link function to be used in the model. For glm this can be a character string naming a family function, a family function or the result of a call to a family function. For glm.fit only the third option is supported. (See family for details of family functions.)
control	a list of parameters controlling the fitting process. See ${\tt brglmControl()}$ for details.
intercept	logical. Should an intercept be included in the null model?
fixed_totals	effective only when family is poisson(). Either NULL (no effect) or a vector that indicates which counts must be treated as a group. See Details for more information and brmultinom().
singular.ok	logical. If FALSE, a singular model is an error.

# Details

A detailed description of the supported adjustments and the quasi Fisher scoring iteration is given in the iteration vignette (see, vignette("iteration", "brglm2") or Kosmidis et al, 2020). A shorter description of the quasi Fisher scoring iteration is also given in one of the vignettes of the *enrichwith* R package (see, https://cran.r-project.org/package=enrichwith/vignettes/ bias.html). Kosmidis and Firth (2010) describe a parallel quasi Newton-Raphson iteration with the same stationary point.

In the special case of generalized linear models for binomial, Poisson and multinomial responses, the adjusted score equation approaches for type = "AS\_mixed", type = "AS\_mean", and type = "AS\_median" (see below for what methods each type corresponds) return estimates with improved frequentist properties, that are also always finite, even in cases where the maximum likelihood estimates are infinite (e.g. complete and quasi-complete separation in multinomial regression). See, Kosmidis and Firth (2021) for a proof for binomial-response GLMs with Jeffreys-prior penalties to the log-likelihood, which is equivalent to mean bias reduction for logistic regression. See, also, detectseparation::detect\_separation() and detectseparation::check\_infinite\_estimates() for pre-fit and post-fit methods for the detection of infinite estimates in binomial response generalized linear models.

The type of score adjustment to be used is specified through the type argument (see brglmControl() for details). The available options are

- type = "AS\_mixed": the mixed bias-reducing score adjustments in Kosmidis et al (2020) that result in mean bias reduction for the regression parameters and median bias reduction for the dispersion parameter, if any; default.
- type = "AS\_mean": the mean bias-reducing score adjustments in Firth, 1993 and Kosmidis & Firth, 2009. type = "AS\_mixed" and type = "AS\_mean" will return the same results when family is binomial() or poisson(), i.e. when the dispersion is fixed
- type = "AS\_median": the median bias-reducing score adjustments in Kenne Pagui et al. (2017)
- type = "MPL\_Jeffreys": maximum penalized likelihood with powers of the Jeffreys prior as penalty.
- type = "ML": maximum likelihood.
- type = "correction": asymptotic bias correction, as in Cordeiro & McCullagh (1991).

The null deviance is evaluated based on the fitted values using the method specified by the type argument (see brglmControl()).

The family argument of the current version of brglmFit() can accept any combination of "family" objects and link functions, including families with user-specified link functions, mis() links, and power() links, but excluding quasi(), quasipoisson() and quasibinomial() families.

The description of method argument and the Fitting functions section in glm() gives information on supplying fitting methods to glm().

fixed\_totals specifies groups of observations for which the sum of the means of a Poisson model will be held fixed to the observed count for each group. This argument is used internally in brmultinom() and bracl() for baseline-category logit models and adjacent category logit models, respectively.

brglm\_fit() is an alias to brglmFit().

## Author(s)

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#### brglmFit

#### References

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, **108**, 71-82. doi:10.1093/biomet/asaa052.

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Cordeiro G M, McCullagh P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B (Methodological)*, **53**, 629-643. doi:10.1111/j.25176161.1991.tb01852.x.

Firth D (1993). Bias reduction of maximum likelihood estimates. *Biometrika*. **80**, 27-38. doi:10.2307/2336755.

Kenne Pagui E C, Salvan A, Sartori N (2017). Median bias reduction of maximum likelihood estimates. *Biometrika*, **104**, 923–938. doi:10.1093/biomet/asx046.

Kosmidis I, Firth D (2009). Bias reduction in exponential family nonlinear models. *Biometrika*, **96**, 793-804. doi:10.1093/biomet/asp055.

Kosmidis I, Firth D (2010). A generic algorithm for reducing bias in parametric estimation. *Electronic Journal of Statistics*, **4**, 1097-1112. doi:10.1214/10EJS579.

Kosmidis I (2014). Bias in parametric estimation: reduction and useful side-effects. *WIRE Computational Statistics*, **6**, 185-196. doi:10.1002/wics.1296.

#### See Also

brglmControl(), glm.fit(), glm()

## Examples

```
## The lizards example from ?brglm::brglm
data("lizards")
# Fit the model using maximum likelihood
lizardsML <- glm(cbind(grahami, opalinus) ~ height + diameter +</pre>
                 light + time, family = binomial(logit), data = lizards,
                 method = "glm.fit")
# Mean bias-reduced fit:
lizardsBR_mean <- glm(cbind(grahami, opalinus) ~ height + diameter +</pre>
                      light + time, family = binomial(logit), data = lizards,
                      method = "brglmFit")
# Median bias-reduced fit:
lizardsBR_median <- glm(cbind(grahami, opalinus) ~ height + diameter +
                        light + time, family = binomial(logit), data = lizards,
                        method = "brglmFit", type = "AS_median")
summary(lizardsML)
summary(lizardsBR_median)
summary(lizardsBR_mean)
# Maximum penalized likelihood with Jeffreys prior penatly
lizards_Jeffreys <- glm(cbind(grahami, opalinus) ~ height + diameter +</pre>
                        light + time, family = binomial(logit), data = lizards,
                        method = "brglmFit", type = "MPL_Jeffreys")
# lizards_Jeffreys is the same fit as lizardsBR_mean (see Firth, 1993)
all.equal(coef(lizardsBR_mean), coef(lizards_Jeffreys))
```

```
# Maximum penalized likelihood with powers of the Jeffreys prior as
# penalty. See Kosmidis & Firth (2021) for the finiteness and
# shrinkage properties of the maximum penalized likelihood
# estimators in binomial response models
a <- seq(0, 20, 0.5)
coefs <- sapply(a, function(a) {</pre>
      out <- glm(cbind(grahami, opalinus) ~ height + diameter +</pre>
             light + time, family = binomial(logit), data = lizards,
             method = "brglmFit", type = "MPL_Jeffreys", a = a)
      coef(out)
})
# Illustration of shrinkage as a grows
matplot(a, t(coefs), type = "l", col = 1, lty = 1)
abline(0, 0, col = "grey")
## Another example from
## King, Gary, James E. Alt, Nancy Elizabeth Burns and Michael Laver
## (1990). "A Unified Model of Cabinet Dissolution in Parliamentary
## Democracies", _American Journal of Political Science_, **34**, 846-870
data("coalition", package = "brglm2")
# The maximum likelihood fit with log link
coalitionML <- glm(duration ~ fract + numst2, family = Gamma, data = coalition)
# The mean bias-reduced fit
coalitionBR_mean <- update(coalitionML, method = "brglmFit")</pre>
# The bias-corrected fit
coalitionBC <- update(coalitionML, method = "brglmFit", type = "correction")</pre>
# The median bias-corrected fit
coalitionBR_median <- update(coalitionML, method = "brglmFit", type = "AS_median")</pre>
## An example with offsets from Venables & Ripley (2002, p.189)
data("anorexia", package = "MASS")
anorexML <- glm(Postwt ~ Prewt + Treat + offset(Prewt),</pre>
                family = gaussian, data = anorexia)
anorexBC <- update(anorexML, method = "brglmFit", type = "correction")</pre>
anorexBR_mean <- update(anorexML, method = "brglmFit")</pre>
anorexBR_median <- update(anorexML, method = "brglmFit", type = "AS_median")
# All methods return the same estimates for the regression
# parameters because the maximum likelihood estimator is normally
# distributed around the `true` value under the model (hence, both
# mean and component-wise median unbiased). The Wald tests for
# anorexBC and anorexBR_mean differ from anorexML because the
# bias-reduced estimator of the dispersion is the unbiased, by
# degree of freedom adjustment (divide by n - p), estimator of the
# residual variance. The Wald tests from anorexBR_median are based
```

## brmultinom

```
# on the median bias-reduced estimator of the dispersion that
# results from a different adjustment of the degrees of freedom
# (divide by n - p - 2/3)
summary(anorexML)
summary(anorexBC)
summary(anorexBR_mean)
summary(anorexBR_median)
## endometrial data from Heinze & Schemper (2002) (see ?endometrial)
data("endometrial", package = "brglm2")
endometrialML <- glm(HG ~ NV + PI + EH, data = endometrial,</pre>
                     family = binomial("probit"))
endometrialBR_mean <- update(endometrialML, method = "brglmFit",
                              type = "AS_mean")
endometrialBC <- update(endometrialML, method = "brglmFit",</pre>
                        type = "correction")
endometrialBR_median <- update(endometrialML, method = "brglmFit",</pre>
                                type = "AS_median")
summary(endometrialML)
summary(endometrialBC)
summary(endometrialBR_mean)
summary(endometrialBR_median)
```

brmultinom	Bias reduction for multinomial response models using the Poisson
	trick.

# Description

brmultinom() is a wrapper of brglmFit() that fits multinomial regression models using implicit and explicit bias reduction methods. See Kosmidis & Firth (2011) for details.

#### Usage

```
brmultinom(
  formula,
  data,
  weights,
  subset,
  na.action,
  contrasts = NULL,
  ref = 1,
  model = TRUE,
  x = TRUE,
  control = list(...),
  ...
)
```

### Arguments

formula	a formula expression as for regression models, of the form response ~ predictors. The response should be a factor or a matrix with K columns, which will be interpreted as counts for each of K classes. A log-linear model is fitted, with coefficients zero for the first class. An offset can be included: it should be a numeric matrix with K columns if the response is either a matrix with K columns or a factor with K >= 2 classes, or a numeric vector for a response factor with 2 levels. See the documentation of formula() for other details.
data	an optional data frame in which to interpret the variables occurring in formula.
weights	optional case weights in fitting.
subset	expression saying which subset of the rows of the data should be used in the fit. All observations are included by default.
na.action	a function to filter missing data.
contrasts	a list of contrasts to be used for some or all of the factors appearing as variables in the model formula.
ref	the reference category to use for multinomial regression. Either an integer, in which case levels(response)[ref] is used as a baseline, or a character string. Default is 1.
model	logical. If true, the model frame is saved as component model of the returned object.
x	should the model matrix be included with in the result (default is TRUE).
control	a list of parameters for controlling the fitting process. See brglmControl() for details.
	arguments to be used to form the default control argument if it is not supplied directly.

## Details

The models brmultinom() handles are also known as baseline-category logit models (see, Agresti, 2002, Section 7.1), because they model the log-odds of every category against a baseline category. The user can control which baseline (or reference) category is used via the ref. By default brmultinom() uses the first category as reference.

The maximum likelihood estimates for the parameters of baseline-category logit models have infinite components with positive probability, which can result in problems in their estimation and the use of inferential procedures (e.g. Wad tests). Albert and Andreson (1984) have categorized the possible data patterns for such models into the exclusive and exhaustive categories of complete separation, quasi-complete separation and overlap, and showed that infinite maximum likelihood estimates result when complete or quasi-complete separation occurs.

The adjusted score approaches to bias reduction that brmultinom() implements for type = "AS\_mean" and type = "AS\_median" are alternatives to maximum likelihood that result in estimates with smaller asymptotic mean and median bias, respectively, that are also *always* finite, even in cases of complete or quasi-complete separation.

brmultinom() is a wrapper of brglmFit() that fits multinomial logit regression models through the 'Poisson trick' (see, for example, Palmgren, 1981; Kosmidis & Firth, 2011).

#### brmultinom

The implementation relies on the construction of an extended model matrix for the log-linear model and constraints on the sums of the Poisson means. Specifically, a log-linear model is fitted on a Kronecker product of the original model matrix X implied by the formula, augmented by nrow(X) dummy variables.

The extended model matrix is sparse, and the **Matrix** package is used for its effective storage.

While brmultinom() can be used for analyses using multinomial regression models, the current implementation is more of a proof of concept and is not expected to scale well with either of nrow(X), ncol(X) or the number of levels in the categorical response.

#### Author(s)

Ioannis Kosmidis [aut, cre] <ioannis.kosmidis@warwick.ac.uk>

#### References

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Agresti A (2002). *Categorical data analysis* (2nd edition). Wiley Series in Probability and Statistics. Wiley.

Albert A, Anderson J A (1984). On the Existence of Maximum Likelihood Estimates in Logistic Regression Models. *Biometrika*, **71** 1–10. doi:10.2307/2336390.

Kosmidis I, Firth D (2011). Multinomial logit bias reduction via the Poisson log-linear model. *Biometrika*, **98**, 755-759. doi:10.1093/biomet/asr026.

Palmgren, J (1981). The Fisher Information Matrix for Log Linear Models Arguing Conditionally on Observed Explanatory Variables. *Biometrika*, **68**, 563-566. doi:10.1093/biomet/68.2.563.

#### See Also

nnet::multinom(), bracl() for adjacent category logit models for ordinal responses

## Examples

## The housing data analysis from ?MASS::housing

```
all.equal(fitted(houseML3), fitted(houseML1), tolerance = 1e-10)
# Bias reduction
houseBR3 <- update(houseML3, type = "AS_mean")</pre>
# Bias correction
houseBC3 <- update(houseML3, type = "correction")</pre>
## Reproducing Bull et al. (2002, Table 3)
data("hepatitis", package = "brglm2")
# Construct a variable with the multinomial categories according to
# the HCV and nonABC columns
hepat <- hepatitis</pre>
hepat$type <- with(hepat, factor(1 - HCV * nonABC + HCV + 2 * nonABC))</pre>
hepat$type <- factor(hepat$type, labels = c("noDisease", "C", "nonABC"))</pre>
contrasts(hepat$type) <- contr.treatment(3, base = 1)</pre>
# Maximum likelihood estimation fails to converge because some estimates are infinite
hepML <- brmultinom(type ~ group * time, data = hepat, weights = counts, type = "ML", slowit = 0.1)
# Mean bias reduction returns finite estimates
hep_meanBR <- brmultinom(type ~ group * time, data = hepat, weights = counts, type = "AS_mean")</pre>
# The estimates in Bull et al. (2002, Table 3, DOI: 10.1016/S0167-9473(01)00048-2)
coef(hep_meanBR)
# Median bias reduction also returns finite estimates, which are a bit larger in absolute value
hep_medianBR <- brmultinom(type ~ group * time, data = hepat, weights = counts, type = "AS_median")
coef(hep_medianBR)
```

brnb

Bias reduction for negative binomial regression models

#### Description

brnb() is a function that fits negative binomial regression models using implicit and explicit bias reduction methods.

#### Usage

```
brnb(
  formula,
  data,
  subset,
  weights = NULL,
  offset = NULL,
  link = "log",
  start = NULL,
  etastart = NULL,
```

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```
mustart = NULL,
control = list(...),
na.action,
model = TRUE,
x = FALSE,
y = TRUE,
contrasts = NULL,
intercept = TRUE,
singular.ok = TRUE,
...
```

# Arguments

formula	an object of class "formula" (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under 'Details'.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula), typically the environment from which glm is called.
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of 'prior weights' to be used in the fitting process. Should be NULL or a numeric vector.
offset	this can be used to specify an <i>a priori</i> known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases. One or more offset terms can be included in the formula instead or as well, and if more than one is specified their sum is used. See model.offset.
link	The link function. Currently must be one of "log", "sqrt" or "identity".
start	starting values for the parameters in the linear predictor.
etastart	starting values for the linear predictor.
mustart	starting values for the vector of means.
control	a list of parameters for controlling the fitting process. See brglmControl() for details.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the na.action setting of options, and is na.fail if that is unset. The 'factory-fresh' default is na.omit. Another possible value is NULL, no action. Value na.exclude can be useful.
model	a logical value indicating whether <i>model frame</i> should be included as a component of the returned value.
х, у	For glm: logical values indicating whether the response vector and model matrix used in the fitting process should be returned as components of the returned value.

	For glm.fit: x is a design matrix of dimension $n * p$ , and y is a vector of observations of length n.
contrasts	an optional list. See the contrasts.arg of model.matrix.default.
intercept	logical. Should an intercept be included in the null model?
singular.ok	logical; if FALSE a singular fit is an error.
	For glm: arguments to be used to form the default control argument if it is not supplied directly.
	For weights: further arguments passed to or from other methods.

#### Details

A detailed description of the fitting procedure is given in the iteration vignette (see, vignette("iteration", "brglm2") and Kosmidis et al, 2020). The number of iterations when estimating parameters are controlled by the maxit argument of brglmControl().

The type of score adjustment to be used is specified through the type argument (see brglmControl() for details).

The available options are:

- type = "AS\_mixed": the mixed bias-reducing score adjustments in Kosmidis et al (2020) that
  result in mean bias reduction for the regression parameters and median bias reduction for the
  dispersion parameter, if any; default.
- type = "AS\_mean": the mean bias-reducing score adjustments in Firth (1993) and Kosmidis & Firth (2009).
- type = "AS\_median": the median bias-reducing score adjustments in Kenne Pagui et al. (2017)
- type = "MPL\_Jeffreys": maximum penalized likelihood with powers of the Jeffreys prior as penalty.
- type = "ML": maximum likelihood.
- type = "correction": asymptotic bias correction, as in Cordeiro & McCullagh (1991).

The choice of the parameterization for the dispersion is controlled by the transformation argument (see brglmControl() for details). The default is "identity". Using transformation = "inverse" uses the dispersion parameterization that MASS::glm.nb() uses.

# Value

A fitted model object of class "brnb" inheriting from "negbin" and "brglmFit". The object is similar to the output of brglmFit() but contains four additional components: theta for the maximum likelihood estimate of the dispersion parameter as in MASS::glm.nb(), vcov.mean for the estimated variance-covariance matrix of the regression coefficients, vcov.dispersion for the estimated variance of the dispersion parameter in the chosen parameterization (using the expected information), and twologlik for twice the log-likelihood function.

# Author(s)

Euloge Clovis Kenne Pagui [aut] <kenne@stat.unipd.it>, Ioannis Kosmidis [aut, cre] <ioannis.kosmidis@warwick

# References

Cordeiro G M, McCullagh P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B (Methodological)*, **53**, 629-643. doi:10.1111/j.25176161.1991.tb01852.x.

Firth D (1993). Bias reduction of maximum likelihood estimates. *Biometrika*. **80**, 27-38. doi:10.2307/2336755.

Kenne Pagui E C, Salvan A, Sartori N (2017). Median bias reduction of maximum likelihood estimates. *Biometrika*, **104**, 923–938. doi:10.1093/biomet/asx046.

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Kosmidis I, Firth D (2009). Bias reduction in exponential family nonlinear models. *Biometrika*, **96**, 793-804. doi:10.1093/biomet/asp055.

#### Examples

```
## Example in Saha, K., & Paul, S. (2005). Bias-corrected maximum
## likelihood estimator of the negative binomial dispersion
## parameter. Biometrics, 61, 179--185.
#
# Number of revertant colonies of salmonella data
salmonella <- data.frame(freg = c(15, 16, 16, 27, 33, 20,</pre>
                                    21, 18, 26, 41, 38, 27,
                                    29, 21, 33, 60, 41, 42),
                          dose = rep(c(0, 10, 33, 100, 333, 1000), 3),
                          observation = rep(1:3, each = 6))
# Maximum likelihood fit with glm.nb of MASS
salmonella_fm <- freq ~ dose + log(dose + 10)</pre>
fitML_glmnb <- MASS::glm.nb(salmonella_fm, data = salmonella)</pre>
# Maximum likelihood fit with brnb
fitML <- brnb(salmonella_fm, data = salmonella,</pre>
              link = "log", transformation = "inverse", type = "ML")
# Mean bias-reduced fit
fitBR_mean <- update(fitML, type = "AS_mean")</pre>
# Median bias-reduced fit
fitBR_median <- update(fitML, type = "AS_median")</pre>
# Mixed bias-reduced fit
fitBR_mixed <- update(fitML, type = "AS_mixed")</pre>
# Mean bias-corrected fit
fitBC_mean <- update(fitML, type = "correction")</pre>
# Penalized likelihood with Jeffreys-prior penalty
fit_Jeffreys <- update(fitML, type = "MPL_Jeffreys")</pre>
# The parameter estimates from glm.nb and brnb with type = "ML" are
# numerically the same
```

brnb

```
coalition
```

```
all.equal(c(coef(fitML_glmnb), fitML_glmnb$theta),
            coef(fitML, model = "full"), check.attributes = FALSE)
# Because of the invariance properties of the maximum likelihood,
# median reduced-bias, and mixed reduced-bias estimators the
# estimate of a monotone function of the dispersion should be
# (numerically) the same as the function of the estimate of the
# dispersion:
# ML
coef(fitML, model = "dispersion")
1 / coef(update(fitML, transformation = "identity"), model = "dispersion")
# Median BR
coef(fitBR_median, model = "dispersion")
1 / coef(update(fitBR_median, transformation = "identity"), model = "dispersion")
# Mixed BR
coef(fitBR_mixed, model = "dispersion")
1 / coef(update(fitBR_mixed, transformation = "identity"), model = "dispersion")
## The same is not true for mean BR
coef(fitBR_mean, model = "dispersion")
1 / coef(update(fitBR_mean, transformation = "identity"), model = "dispersion")
## An example from Venables & Ripley (2002, p.169).
data("quine", package = "MASS")
quineML <- brnb(Days ~ Sex/(Age + Eth*Lrn), link = "sqrt", transformation="inverse",</pre>
                data = quine, type="ML")
quineBR_mean <- update(quineML, type = "AS_mean")</pre>
quineBR_median <- update(quineML, type = "AS_median")</pre>
quineBR_mixed <- update(quineML, type = "AS_mixed")</pre>
quine_Jeffreys <- update(quineML, type = "MPL_Jeffreys")</pre>
fits <- list(ML = quineML,</pre>
             AS_mean = quineBR_mean,
             AS_median = quineBR_median,
             AS_mixed = quineBR_mixed,
             MPL_Jeffreys = quine_Jeffreys)
sapply(fits, coef, model = "full")
```

coalition

Coalition data

#### Description

This data set contains survival data on government coalitions in parliamentary democracies (Belgium, Canada, Denmark, Finland, France, Iceland, Ireland, Israel, Italy, Netherlands, Norway, Por-

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# coef.brglmFit

tugal, Spain, Sweden, and the United Kingdom) for the period 1945-1987. For parsimony, country indicator variables are omitted in the sample data.

#### Usage

coalition

# Format

A data frame with 314 rows and the 7 variables "duration", "ciep12", "invest", "fract", "polar", "numst2", and "crisis". For variable descriptions, please refer to King et al (1990).

# Note

Data is as it is provided by the Zeilig R package.

#### References

King G, Alt J E, Burns N E, Laver M. (1990). A Unified Model of Cabinet Dissolution in Parliamentary Democracies. *American Journal of Political Science*, **34**, 846-870. doi:10.2307/2111401.

King G, Alt J E, Burns N E, Laver M. ICPSR Publication Related Archive, 1115.

## See Also

brglm\_fit()

coef.brglmFit Extract model coefficients from "brglmFit" objects

#### Description

Extract model coefficients from "brglmFit" objects

## Usage

```
## S3 method for class 'brglmFit'
coef(object, model = c("mean", "full", "dispersion"), ...)
```

## Arguments

object	an object for which the extraction of model coefficients is meaningful.
model	one of "mean" (default), "dispersion", "full", to return the estimates of the parameters in the linear prediction only, the estimate of the dispersion parameter only, or both, respectively.
	other arguments.

# Details

See coef() for more details.

## See Also

coef()

coef.brglmFit\_expo Extract estimates from "brglmFit\_expo" objects

# Description

Extract estimates from "brglmFit\_expo" objects

## Usage

## S3 method for class 'brglmFit\_expo'
coef(object, ...)

# Arguments

object	an object for which the extraction of model coefficients is meaningful.
	other arguments.

# Description

Extract model coefficients from "brnb" objects

# Usage

```
## S3 method for class 'brnb'
coef(object, model = c("mean", "full", "dispersion"), ...)
```

# Arguments

object ar	n object for which the extraction of model coefficients is meaningful.
model or	ne of "mean" (default), "full", "dispersion", to return the estimates of the
pa	arameters in the linear prediction only, or both, the estimate of the dispersion
pa	arameter only, respectively.
ot	her arguments.

# Details

See coef() for more details.

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 $\verb|confint.brglmFit||$ 

Method for computing confidence intervals for one or more regression parameters in a "brglmFit" object

# Description

Method for computing confidence intervals for one or more regression parameters in a "brglmFit" object

#### Usage

```
## S3 method for class 'brglmFit'
confint(object, parm, level = 0.95, ...)
```

# Arguments

object	a fitted model object.
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are consid- ered.
level	the confidence level required.
	additional argument(s) for methods.

confint.brmultinom	Method for computing confidence intervals for one or more regression
	parameters in a "brmultinom" object

# Description

Method for computing confidence intervals for one or more regression parameters in a "brmultinom" object

## Usage

```
## S3 method for class 'brmultinom'
confint(object, parm, level = 0.95, ...)
```

### Arguments

object	a fitted model object.
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are consid- ered.
level	the confidence level required.
	additional argument(s) for methods.

confint.brnb

# Description

Method for computing Wald confidence intervals for one or more regression parameters in a "brnb" object

#### Usage

```
## S3 method for class 'brnb'
confint(object, parm, level = 0.95, ...)
```

## Arguments

object	a fitted model object.
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are consid- ered.
level	the confidence level required.
	additional argument(s) for methods.

endometrial	Histology grade and risk factors for 79 cases of endometrial cancer
-------------	---

# Description

Histology grade and risk factors for 79 cases of endometrial cancer

## Usage

endometrial

## Format

A data frame with 79 rows and 4 variables:

- NV: neovasculization with coding 0 for absent and 1 for present
- PI: pulsality index of arteria uterina
- EH: endometrium height
- HG histology grade with coding 0 for low grade and 1 for high grade

#### enzymes

## Source

The packaged data set was downloaded in .dat format from https://users.stat.ufl.edu/~aa/glm/data/. The latter link provides the data sets used in Agresti (2015).

The endometrial data set was first analyzed in Heinze and Schemper (2002), and was originally provided by Dr E. Asseryanis from the Medical University of Vienna.

## References

Agresti A (2015). *Foundations of Linear and Generalized Linear Models*. Wiley Series in Probability and Statistics. Wiley.

Heinze G, Schemper M (2002). A Solution to the Problem of Separation in Logistic Regression. *Statistics in Medicine*, **21**, 2409–2419. doi:10.1002/sim.1047.

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, **108**, 71-82. doi:10.1093/biomet/asaa052.

#### See Also

brglm\_fit()

enzymes

Liver Enzyme Data

#### Description

Liver enzyme data collected from 218 patients with liver disease (Plomteux, 1980). The laboratory profile consists of enzymatic activity measured for four liver enzymes: aspartate aminotransferase (AST), alanine aminotransferase (ALT), glutamate dehydrogenase (GLDH) and ornithine carbamyl-transferase (OCT).

#### Usage

enzymes

## Format

A data frame with 218 rows and the following 6 columns:

- Patient: Patient ID
- Group: Four diagnostic groups were considered: acute viral hepatitis (1), persistent chronic hepatitis (2), aggressive chronic hepatitis (3) and post-necrotic cirrhosis (4).
- AST: Aspartate aminotransferase (in U/L)
- ALT: Alanine aminotransferase (in U/L)
- GLDH: Glutamate dehydrogenase (in U/L)
- OCT: Ornithine carbamyltransferase (in U/L)

#### Source

Data from Albert and Harris (1984, Chapter 5, Appendix I), and is also provided by the **pmlr** R package.

#### References

Albert A, Harris E K (1984). *Multivariate Interpretation of Clinical Laboratory Data*. Dekker: New York.

Plomteux G (1980). Multivariate analysis of an enzyme profile for the differential diagnosis of viral hepatitis. *Clinical Chemistry*, **26**, 1897-1899.

expo.brglmFit

*Estimate the exponential of parameters of generalized linear models using various methods* 

#### Description

The expo() method uses the supplied "brglmFit" or "glm" object to estimate the exponential of parameters of generalized linear models with maximum likelihood or various mean and median bias reduction methods. expo() is useful for computing (corrected) estimates of the multiplicative impact of a unit increase on a covariate on the mean of a Poisson log-linear model (family = poisson("log") in glm()) while adjusting for other covariates, the odds ratio associated with a unit increase on a covariate in a logistic regression model (family = binomial("logit") in glm()) while adjusting for other covariated with a unit increase on a covariate, the relative risk associated with a unit increase on a covariate in a relative risk regression model (family = binomial("log") in glm()) while adjusting for other covariates, the relative risk associated with a unit increase on a covariate in a relative risk regression model (family = binomial("log") in glm()) while adjusting for other covariates, the relative risk associated with a unit increase on a covariate in a relative risk regression model (family = binomial("log") in glm()) while adjusting for other covariates, the relative risk associated with a unit increase on a covariate in a relative risk regression model (family = binomial("log") in glm()) while adjusting for other covariates, among others.

#### Usage

```
## S3 method for class 'brglmFit'
expo(
    object,
    type = c("correction*", "correction+", "Lylesetal2012", "AS_median", "ML"),
    level = 0.95
)
## S3 method for class 'glm'
expo(
    object,
    type = c("correction*", "correction+", "Lylesetal2012", "AS_median", "ML"),
    level = 0.95
)
```

#### expo.brglmFit

#### Arguments

object	an object of class "brglmFit" or "glm".
type	the type of correction to be used. The available options are "correction*" (ex- plicit mean bias correction with a multiplicative adjustment), "correction*" (explicit mean bias correction with an additive adjustment), "Lylesetal2012" (explicit median bias correction using the multiplicative adjustment in Lyles et al., 2012), "AS_median" (median bias reduction), and "ML" (maximum likeli- hood). See Details.
level	the confidence level required. Default is 0.95.

## Details

The supported methods through the type argument are:

- "ML": the estimates of the exponentiated parameters are  $\exp(\hat{\theta}_j)$ , where  $\theta_j$  is the maximum likelihood estimates for the *j*th regression parameter.
- "correction\*": the estimates of the exponentiated parameters are  $\exp(\hat{\theta}_j)/(1+\hat{v}_j/2)$ , where  $\hat{\theta}_j$  is the estimate of the *j*th regression parameter using type = "AS\_mixed" in brglmFit().
- "correction+": the estimates of the exponentiated parameters are  $\exp(\hat{\theta}_j)(1-\hat{v}_j/2)$ , where  $\hat{\theta}_j$  is the estimate of the *j*th regression parameter using type = "AS\_mixed" in brglmFit().
- "Lylesetal2012": the estimates of the exponentiated parameters are  $\exp(\hat{\theta}_j)exp(-\hat{v}_j/2)$ , where  $\hat{\theta}_j$  is the estimate of the *j*th regression parameter using type = "AS\_mixed" in brglmFit(). This estimator has been proposed in Lyles et al. (2012).
- "AS\_median": the estimates of the exponentiated parameters are  $\exp(\hat{\theta}_j)$ , where  $\hat{\theta}_j$  is the estimate of the *j*th regression parameter using type = "AS\_median" in brglmFit().

"correction\*" and "correction+" are based on multiplicative and additive adjustments, respectively, of the exponential of a reduced-bias estimator (like the ones coming from brglmFit() with type = "AS\_mixed", type = "AS\_mean", and type = "correction"). The form of those adjustments results from the expression of the first-term in the mean bias expansion of the exponential of a reduced-bias estimator. See, for example, Di Caterina & Kosmidis (2019, expression 12) for the general form of the first-term of the mean bias of a smooth transformation of a reduced-bias estimator.

The estimators from "correction+", "correction\*", "Lylesetal2012" have asymptotic mean bias of order smaller than than of the maximum likelihood estimator. The estimators from "AS\_median" are asymptotically closed to being median unbiased than the maximum likelihood estimator is.

Estimated standard errors are computed using the delta method, where both the Jacobin and the information matrix are evaluated at the logarithm of the estimates of the exponentiated parameters.

Confidence intervals results by taking the exponential of the limits of standard Wald-type intervals computed at the logarithm of the estimates of the exponentiated parameters.

#### Value

a list inheriting from class "brglmFit\_expo" with components coef (the estimates of the exponentiated regression parameters), se (the corresponding estimated standard errors for the exponentiated parameters), ci (confidence intervals of level level for the exponentiated parameters), and type for the type of correction that has been requested.

#### Author(s)

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# References

Di Caterina C, Kosmidis I (2019). Location-Adjusted Wald Statistics for Scalar Parameters. *Computational Statistics & Data Analysis*, **138**, 126-142. doi:10.1016/j.csda.2019.04.004.

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59. doi:10.1007/s11222019098606.

Cordeiro G M, McCullagh P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B (Methodological)*, **53**, 629-643. doi:10.1111/j.25176161.1991.tb01852.x.

Lyles R H, Guo Y, Greenland S (2012). Reducing bias and mean squared error associated with regression-based odds ratio estimators. *Journal of Statistical Planning and Inference*, **142** 3235–3241. doi:10.1016/j.jspi.2012.05.005.

## See Also

brglm\_fit() and and brglm\_control()

## Examples

```
## The lizards example from ?brglm::brglm
lizardsML <- glm(cbind(grahami, opalinus) ~ height + diameter +</pre>
                 light + time, family = binomial(logit), data = lizards,
                 method = "glm.fit")
# Get estimates, standard errors, and confidence intervals of odds
# ratios with various methods
expo(lizardsML, type = "ML")
expo(lizardsML, type = "correction*")
expo(lizardsML, type = "Lylesetal2012")
expo(lizardsML, type = "correction+")
expo(lizardsML, type = "AS_median")
## Example from ?glm
## Dobson (1990) Page 93: Randomized Controlled Trial :
counts <- c(18,17,15,20,10,20,25,13,12)
outcome <- gl(3, 1, 9)
treatment <- gl(3,3)</pre>
glm.D93 <- glm(counts ~ outcome + treatment, family = poisson())</pre>
expo(glm.D93, type = "correction*")
```

hepatitis

Post-transfusion hepatitis: impact of non-A, non-B hepatitis surrogate tests

#### hepatitis

### Description

Data from a randomized double-blind trial to assess whether withholding donor blood positive for the non-A, non-B ("NANB") surrogate markers would reduce the frequency of post-transfusion hepatitis. The dataset contains 4588 subjects enrolled from 1988 to 1992 into two study groups that received allogenic blood from which units positive for NANB surrogate markers were withheld (n = 2311) or not withheld (n = 2277). Subjects were followed up for 6 months and assessed for the presence of post-transfusion hepatitis.

## Usage

hepatitis

#### Format

A data frame with 28 rows and the following 6 columns:

- city: Subjects were recruited from 3 Canadian Red Cross Society Blood Centres and 13 university-affiliated hospitals in 3 cities: Toronto, Hamilton and Winnipeg.
- group: Eligible subjects were assigned to one of two allogenic blood recipient groups. One group received products that had only routine Canadian transfusion-transmissible disease marker screening (no-withhold). The other group received only products that were not positive for NANB surrogate markers (withhold).
- time: Hepatitis C (HCV) screening was introduced in Canada in May, 1990. Subjects were recruited into the study before (pre) and after (post) the introduction of anti-HCV testing.
- HCV: Post-transfusion HCV hepatitis present (1) or absent (0).
- nonABC: Post-transfusion non-A, non-B, non-C hepatitis present (1) or absent (0)
- counts: Number of subjects

#### Source

Data is from Blajchman et al. (1995), also analyzed in Bull et al. (2002), and is also provided by the **pmlr** R package.

#### References

Bull S B, Mak C, Greenwood C M T (2002). A modified score function estimator for multinomial logistic regression in small samples. *Computational Statistics & Data Analysis*, **39**, 57-74. doi:10.1016/S01679473(01)000482

Blajchman M A, Bull S B and Feinman S V (1995). Post-transfusion hepatitis: impact of non-A, non-B hepatitis surrogate tests. *The Lancet*, **345**, 21–25. doi:10.1016/S01406736(95)911537

lizards

# Description

Habitat preferences of lizards

#### Usage

lizards

# Format

A data frame with 23 rows and 6 columns:

- grahami. count of grahami lizards
- opalinus. count of opalinus lizards
- height. a factor with levels <5ft, >=5ft
- diameter. a factor with levels <=2in, >2in
- light. a factor with levels sunny, shady
- time. a factor with levels early, midday, late

The variables grahami and opalinus are counts of two lizard species at two different perch heights, two different perch diameters, in sun and in shade, at three times of day.

#### Source

McCullagh P, Nelder J A (1989) *Generalized Linear Models* (2nd Edition). London: Chapman and Hall.

Originally from

Schoener T W (1970) Nonsynchronous spatial overlap of lizards in patchy habitats. \_Ecology\_ \*51\*, 408-418.

## See Also

brglm\_fit()

A "link-glm" object for misclassified responses in binomial regression models

# Description

mis() is a "link-glm" object that specifies the link function in Neuhaus (1999, expression (8)) for handling misclassified responses in binomial regression models using maximum likelihood. A prior specification of the sensitivity and specificity is required.

## Usage

```
mis(link = "logit", sensitivity = 1, specificity = 1)
```

## Arguments

link	the baseline link to be used.
sensitivity	the probability of observing a success given that a success actually took place given any covariate values.
specificity	the probability of observing a failure given that a failure actually took place given any covariate values.

# Details

sensitivity + specificity should be greater or equal to 1, otherwise it is implied that the procedure producing the responses performs worse than chance in terms of misclassification.

#### References

Neuhaus J M (1999). Bias and efficiency loss due to misclassified responses in binary regression. Biometrika, **86**, 843-855. https://www.jstor.org/stable/2673589.

#### See Also

glm(), brglm\_fit()

#### Examples

mis

mis

```
## A notable change is coefficients is noted here compared to when
## specificity and sensitity are 1
coef(lizardsML)
coef(lizardsML_mis)
## Bias reduction is also possible
update(lizardsML_mis, method = "brglmFit", type = "AS_mean",
    start = coef(lizardsML))
update(lizardsML_mis, method = "brglmFit", type = "AS_median",
    start = coef(lizardsML))
```

ordinal\_superiority.bracl

Ordinal superiority scores of Agresti and Kateri (2017)

# Description

ordinal\_superiority() is a method for the estimation and inference about model-based ordinal superiority scores introduced in Agresti and Kateri (2017, Section 5) from fitted objects. The mean bias of the estimates of the ordinal superiority scores can be corrected.

## Usage

```
## S3 method for class 'bracl'
ordinal_superiority(
   object,
   formula,
   data,
   measure = c("gamma", "Delta"),
   level = 0.95,
   bc = FALSE
)
```

## Arguments

object	a fitted object from an ordinal regression model. Currently only models from class "bracl" are supported.
formula	a RHS formula indicating the group variable to use.
data	an optional data frame in which to look for variables with which to compute ordinal superiority measures. If omitted, an attempt is made to use the data that produced object.
measure	either "gamma" (default) or "Delta", specifying the ordinal superiority measure to be returned.
level	the confidence level required when computing confidence intervals for the ordi- nal superiority measures.

36

#### predict.bracl

bc

logical. If FALSE (default) then the ordinal superiority measures are computed using the estimates in object. If TRUE then the ordinal superiority measure estimates are corrected for mean bias.

#### References

Agresti, A., Kateri, M. (2017). Ordinal probability effect measures for group comparisons in multinomial cumulative link models. *Biometrics*, **73** 214-219. doi:10.1111/biom.12565.

#### Examples

## End(Not run)

predict.bracl Predict method for bracl fits

## Description

Obtain class and probability predictions from a fitted adjacent category logits model.

#### Usage

```
## S3 method for class 'bracl'
predict(object, newdata, type = c("class", "probs"), ...)
```

#### Arguments

object	a fitted object of class inheriting from "brac1".
newdata	optionally, a data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.
type	the type of prediction required. The default is "class", which produces pre- dictions of the response category at the covariate values supplied in "newdata", selecting the category with the largest probability; the alternative "probs" re- turns all category probabilities at the covariate values supplied in newdata.
	further arguments passed to or from other methods.

## Details

If newdata is omitted the predictions are based on the data used for the fit.

#### Value

If type = "class" a vector with the predicted response categories; if type = "probs" a matrix of probabilities for all response categories at newdata.

## Examples

```
sapply(c("class", "probs"), function(what) predict(fit_bracl_p, newdata, what))
```

predict.brmultinom *Predict method for brmultinom fits* 

#### Description

Obtain class and probability predictions from a fitted baseline category logits model.

## residuals.brmultinom

## Usage

```
## S3 method for class 'brmultinom'
predict(object, newdata, type = c("class", "probs"), ...)
```

#### Arguments

object	a fitted object of class inheriting from "brmultinom".
newdata	optionally, a data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.
type	the type of prediction required. The default is "class", which produces pre- dictions of the response category at the covariate values supplied in "newdata", selecting the category with the largest probability; the alternative "probs" re- turns all category probabilities at the covariate values supplied in newdata.
••••	further arguments passed to or from other methods.

## Details

If newdata is omitted the predictions are based on the data used for the fit.

## Value

If type = "class" a vector with the predicted response categories; if type = "probs" a matrix of probabilities for all response categories at newdata.

#### Examples

data("housing", package = "MASS")

residuals.brmultinom Residuals for multinomial logistic regression and adjacent category logit models

## Description

Residuals for multinomial logistic regression and adjacent category logit models

```
## S3 method for class 'brmultinom'
residuals(object, type = c("pearson", "response", "deviance", "working"), ...)
## S3 method for class 'bracl'
residuals(object, type = c("pearson", "response", "deviance", "working"), ...)
```

#### Arguments

object	the object coming out of bracl() and brmultinom().
type	the type of residuals which should be returned. The options are: "pearson" (default), "response", "deviance", "working". See Details.
	Currently not used.

## Details

The residuals computed are the residuals from the equivalent Poisson log-linear model fit, organized in a form that matches the output of fitted(object, type = "probs"). As a result, the output is residuals defined in terms of the object and expected multinomial counts.

#### See Also

brmultinom bracl

simulate.brmultinom	Method for simulating a data set from "brmultinom" and "bracl"
	objects

#### Description

Method for simulating a data set from "brmultinom" and "bracl" objects

#### Usage

## S3 method for class 'brmultinom'
simulate(object, ...)

#### Arguments

object	an object of class "brmultinom" or "bracl".
	currently not used.

#### Value

A "data.frame" with object\$ncat times the rows that model.frame(object) have and the same variables. If weights has been specified in the call that generated object, then the simulate frequencies will populate the weights variable. Otherwise, the resulting data.frame will have a ".weights" variable with the simulated multinomial counts.

## simulate.brnb

## Examples

```
simulate(stemML1)
```

simulate.brnb Simulate Responses

#### Description

Simulate one or more responses from the distribution corresponding to a fitted model "brnb" object.

#### Usage

## S3 method for class 'brnb'
simulate(object, nsim = 1, seed = NULL, ...)

## Arguments

object	an object representing a fitted model.
nsim	number of response vectors to simulate. Defaults to 1.
seed	an object specifying if and how the random number generator should be initial- ized; see set.seed() for details.
	extra arguments to be passed to methods. Not currently used.

## Examples

```
sim_glmnb <- simulate(fit_glmnb, nsim = 10, seed = 123)
sim_brnb <- simulate(fit_brnb, nsim = 10, seed = 123)
# The results from glm.nb and brnb with type = "ML" are
# exactly the same
all.equal(sim_glmnb, sim_brnb, check.attributes = FALSE)</pre>
```

stemcell

**Opinion on Stem Cell Research and Religious Fundamentalism** 

## Description

A data set from the 2006 General Social Survey that shows the relationship in the United States between opinion about funding stem cell research and the fundamentalism/liberalism of one's religious beliefs, stratified by gender.

#### Usage

stemcell

#### Format

A data frame with 24 rows and 4 variables:

- research: opinion about funding stem cell research with levels definitely, probably, probably not, definitely not
- gender: the gender of the respondent with levels female and male
- religion: the fundamentalism/liberalism of one's religious beliefs with levels fundamentalist, moderate, liberal frequency: the number of times a respondent fell in each of the combinations of levels for research, religion and gender

#### Source

The stemcell data set is analyzed in Agresti (2010, Subsection 4.1.5).

#### References

Agresti A (2010). *Analysis of Ordinal Categorical Data* (2nd edition). Wiley Series in Probability and Statistics. Wiley.

## See Also

bracl()

summary.brglmFit summary() method for brglmFit objects

## Description

summary() method for brglmFit objects

## Usage

```
## S3 method for class 'brglmFit'
summary(
  object,
  dispersion = NULL,
  correlation = FALSE,
  symbolic.cor = FALSE,
  . . .
)
## S3 method for class 'summary.brglmFit'
print(
  х,
 digits = max(3L, getOption("digits") - 3L),
  symbolic.cor = x$symbolic.cor,
  signif.stars = getOption("show.signif.stars"),
  . . .
)
```

#### Arguments

object	an object of class "glm", usually, a result of a call to glm.
dispersion	the dispersion parameter for the family used. Either a single numerical value or NULL (the default), when it is inferred from object (see 'Details').
correlation	logical; if TRUE, the correlation matrix of the estimated parameters is returned and printed.
symbolic.cor	logical. If TRUE, print the correlations in a symbolic form (see symnum) rather than as numbers.
	further arguments passed to or from other methods.
х	an object of class "summary.glm", usually, a result of a call to summary.glm.
digits	the number of significant digits to use when printing.
signif.stars	logical. If TRUE, 'significance stars' are printed for each coefficient.

## Details

The interface of the summary method for "brglmFit" objects is identical to that of "glm" objects. The summary method for "brglmFit" objects computes the p-values of the individual Wald statistics based on the standard normal distribution, unless the family is Gaussian, in which case a t distribution with appropriate degrees of freedom is used.

#### See Also

summary.glm() and glm()

## Examples

## For examples see `examples(brglmFit)`

summary.brnb summary() method for "brnb" objects

## Description

summary() method for "brnb" objects

## Usage

## S3 method for class 'brnb'
summary(object, ...)
## S3 method for class 'summary.brnb'
print(x, digits = max(3, getOption("digits") - 3), ...)

## Arguments

object	an object of class "brnb", typically, a result of a call to brnb().
	further arguments passed to or from other methods.
x	an object of class "summary.brnb", usually, a result of a call to summary.brnb.
digits	the number of significant digits to use when printing.

# Details

The interface of the summary method for "brnb" objects is similar to that of "brglmFit" objects with additional information.

p-values of the individual Wald statistics are based on the standard normal distribution.

# See Also

summary.brglmFit() and glm()

## Examples

# For examples see examples(brnb)

vcov.brglmFit

Return the variance-covariance matrix for the regression parameters in a brglmFit() object

#### Description

Return the variance-covariance matrix for the regression parameters in a brglmFit() object

## Usage

```
## S3 method for class 'brglmFit'
vcov(object, model = c("mean", "full", "dispersion"), complete = TRUE, ...)
```

## Arguments

object	a fitted model object, typically. Sometimes also a summary() object of such a fitted model.
model	character specifying for which component of the model coefficients should be extracted.
complete	for the aov, lm, glm, mlm, and where applicable summary.lm etc methods: log- ical indicating if the full variance-covariance matrix should be returned also in case of an over-determined system where some coefficients are undefined and coef(.) contains NAs correspondingly. When complete = TRUE, vcov() is compatible with coef() also in this singular case.
	additional arguments for method functions. For the glm method this can be used to pass a dispersion parameter.

## Details

The options for model are "mean" for mean regression parameters only (default), "dispersion" for the dispersion parameter (or the transformed dispersion; see brglm\_control()), and "full" for both the mean regression and the (transformed) dispersion parameters.

vcov.brnb

Extract model variance-covariance matrix from "brnb" objects

## Description

Extract model variance-covariance matrix from "brnb" objects

#### Usage

```
## S3 method for class 'brnb'
vcov(object, model = c("mean", "full", "dispersion"), complete = TRUE, ...)
```

# Arguments

object	an object of class "brnb", typically, a result of a call to brnb().
model	character specifying for which component of the model variance-covariance ma- trix should be extracted.
complete	for the aov, lm, glm, mlm, and where applicable summary.lm etc methods: log- ical indicating if the full variance-covariance matrix should be returned also in case of an over-determined system where some coefficients are undefined and coef(.) contains NAs correspondingly. When complete = TRUE, vcov() is compatible with coef() also in this singular case.
	additional arguments for method functions. For the glm method this can be used to pass a dispersion parameter.

# Details

The options for model are "mean" for mean regression only (default), "dispersion" for the dispersion parameter (in a chosen transformation; see brglmControl(), and "full" for both the mean regression and the (transformed) dispersion parameters. See vcov() for more details.

## See Also

vcov()

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